Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Engineering Mathematics-IV

Subject Code : 4TE04EMT1 **Branch: B.Tech (Mech,Auto,EC,EE,EEE,Civil,IC)**

Semester : 4 Date :03/05/2017 Time : 02:00 To 05:00 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q.1 Attempt the following questions: (14)

A) Define: Gradient of the scalar field. (1)

B) If $f(z)=u(x,y)+iv(x,y)$ is analytic then $f'(z)=.....$ (1)

$(a)\frac{\partial u}{\partial x}+i\frac{\partial u}{\partial y}$ $(b)\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$ $(c)\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$ (d) *None of these*

C) True/False: The function $f(z)=\bar{z}$ is not analytic at any point. (1)

D) Write Fourier cosine transform of f(t). (1)

E) State Simpson's 1/3 Rule. (1)

F) True/False: In ODE number of independent variables are more than one. (1)

G) The image of circle $|z-1|=1$ in the complex plane , under the mapping (1)

$w=u+iv=\frac{1}{z}$ is

$(a)|w-1|=1$ $(b)u^2+v^2=1$ $(c)u=\frac{1}{z}$ $(d)v=\frac{1}{z}$

H) The function $u(x,y)=2x-x^2+py^2$ is harmonic if p=…… (1)

(a) 0 (b) 1 (c) 2 (d) 3

I) Define: Solenoidal vector. (1)

J) True/False: In usual notation $\nabla=1-E^{-1}$. (1)

K) Show that $hD=\log(1+\Delta)$. (2)

L) If $F=xz^3i-2x^2yz+2yz^4k$. then find $curl\,F$ at the point $(1,-1,1)$. (2)

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions (14)

A) The following table gives the values of density of saturated water for various temperatures of saturated steam- **(06)**

Temp T (C°)	100	150	200	250	300
Density d (ng/m^3)	958	917	865	799	712

Find by interpolation, the densities when temperatures are (i) 130^0 C & (ii) 275 °C.

B) Using Lagrange's Interpolation ,express $f(x) = \frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as sum of partial fractions. **(04)**

C) Apply Guass Jordan Method to solve the equations $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$ **(04)**

Q.3 Attempt all questions **(14)**

A) Given that - **(05)**

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y=f(x)	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ at x=1.6 .

B) A river is 80 meter wide. The depth 'd' in meters at distance 'x' meters from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's 3/8 Rule. **(05)**

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

C) Using R-K method of fourth order compute y(0.2) & y(0.4) given $\frac{dy}{dx} = y - \frac{2x}{y}; y(0) = 1.$ **(04)**

Q.4 Attempt all questions **(14)**

A) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is Harmonic in some domain & find a harmonic conjugate of u(x,y). **(05)**

B) Using Milne Thompson method determine the analytic function whose imaginary part $v(x, y) = e^x (x\cos y - y\sin y)$ **(05)**

C) Define Mobius transformation. Determine the mobius transformation that maps $z_1 = 0,\ z_2 = 1, z_3 = \infty$ *onto* $w_1 = -1, w_2 = -i, w_3 = 1$ *respectively*. **(04)**

Q.5 Attempt all questions **(14)**

A) Define Directional Derivative of function. Find the Directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x\log z - y^2 = 4$ at $(-1, 2, 1)$. **(05)**

B) Define Curl of a Vector field. Show that A fluid motion is given by $v = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is Irrotational . **(05)**

C) Solve the initial value problem $\frac{dy}{dx} = x\sqrt{y}, y(1) = 1$ & hence find y(1.5) by taking h=0.1 using Euler's method. **(04)**

Q.6 Attempt all questions **(14)**

A) Find Fourier sine transform of $e^{-\frac{ax}{x}}$. **(05)**

B) Find the Fourier cosine transform of e^{-x^2}. **(05)**

C) Check Whether $f(z)=\cos z$ is analytic or not. If analytic ,find its derivative. **(04)**

Q.7 Attempt all questions **(14)**

A) State Green's Theorem. Verify Green's Theorem for **(09)**

$\oint_c \left[\left(x^2-2xy\right)dx+\left(x^2y+3\right)dy\right]$ Where C is the boundary of the bounded region by the parabola $y=x^2$ & line $y=x$.

B) Define: Line Integral. Find Work done if $\vec{F}=2x^2j+3xyk$ displace a particle in the xy-plane from $(0,0)$ *to* $(1,4)$ *along the curve* $y=4x^2$. **(05)**

Q.8 Attempt all questions **(14)**

A) State Stokes's Theorem. Verify Stokes's theorem for the vector field $\vec{F}=\left(x^2-y^2\right)i+2xyj$ in the rectangular region in the xy-plane bounded by $x=-a, x=a, y=0, y=b.$ **(09)**

B) Define: Divergence. For which value of the component v_3 is $v=e^x\cos yi+e^x\sin yj+v_3k$ is Solenoidal. **(05)**